ANALYTICAL DEVELOPMENT MODEL OF BEE COLONY

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***Summary:** A linear model of bee colony development made in Excel 97 has 13 variables and 13 depending (calculated) input parameters. Birth and death rules are separated and show the colony development round the year. The rule of egg laying is trapezium. Example is based on triangle egg laying rule as a special case of trapezium. The curve of changes for amount of bees corresponds to the experimental results.*

INTRODUCTION

Interest in modelling of changes of bee colony composition is concerned with bee activities analysing and forecasting. Bee physiology and biology facts beard in mind let us find simplified overview about processes in bee colony explaining bee colony development in past and forecasting development and opportunities in future. This kind of model is usable both in beekeepers education and in practical beekeeping to forecast productivity indicators taking in account peculiarities of colonies and season.

Separate elements of bee colonies composition's model are described in literature and the mean difference is in the types of functions used to describe the egg laying of bee queen: non-linear (Mihalik, 1980), McLellan et al., 1980), (De Grandi-Hoffman et al., 1989) and linear (Rowland et al., 1982). There is difference in number of influencing factors as well. Newly found experimental results allow improving the models and building them better understandable and better visible.

Aim of this article is to extend the opportunities of known models in longer period of bee colony development.

PRESUMPTONS AND LIMITATIONS

The model is built for a honey bee *(Apis mellifera)* colony located in climatic conditions similar to Latvian ones. Following limitations are presumed:

- bee colony is healthy and does not swarm;
- there is enough space and food for development of the colony;
- there is one bee queen in colony and each egg develops to a bee;
- brood (bee development stages till the birth) development periods are 3+6+12=21 days (appropriately: egg+larvae+pupa);
- presence of drones is ignored because of small amount in the colony (0.5-5%) (Winston, 1992);
- egg laying by queen is limited by insufficient amount of bees;

DESCRIPTION

Development rule is described in two years – conditionally 1-st and 2-nd years which represent two following beekeeping seasons of the same colony. In the 1-st year we look at the development of wintering bees but in the 2-nd – full year development cycle of a bee colony.
Changes of the number of live bees in a colony are determined by a difference between born and dead bees.

$$X=B-N, \tag{1}$$

where X – number of live bees in colony, B – number of born bees, N – number of dead bees.
To calculate changes of the number of live bees it is necessary to describe bee birth and death rules.

Bee birth rule.

Bee birth is determined by egg laying rule of bee queen because working bee is borning from the egg lied by queen after 21 day. As a general rule of bee queen is assumed trapezium ABCD (Figure 1) which in special cases forms a triangle (ABD) or rectangle AB'C'D.

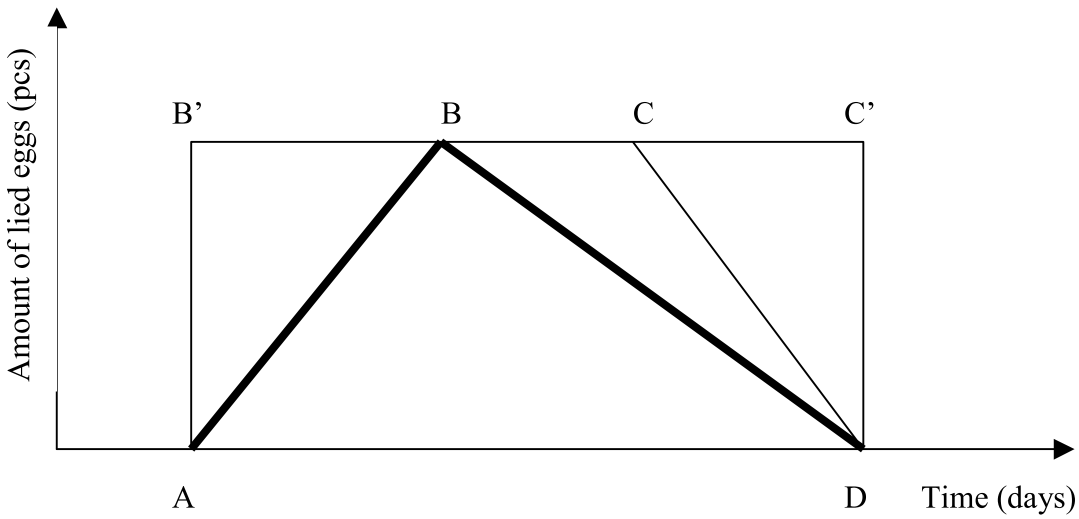

Figure 1. Variants of approximations of bee queen laying curves: AB'C'D – rectangle, ABCD – trapeze and ABD – triangle.

Closer we will look at a case when bee queen egg laying happens accordingly to the triangle rule (Figure 1). There are three characteristic egg laying periods (Figure 2): 1) end of active season of the 1-st year for accumulation of wintering bees ($t_0 \div t_2$); 2) during wintering ($t_3 \div t_4$); and 3) During all the active season of the 2-nd year ($t_4 \div t_8$).

Bee death rule.

There are three characteristic periods of bee death (Figure 3): 1) during wintering ($t_9 \div t_{10}$); 2) during spring development ($t_{10} \div t_{11}$) and 3) during all the active season in the 2-nd year ($t_{12} \div t_{15}$). The last one is similar to the queen egg laying rule delayed by the average length of life +21 day. Exception is the end part where the accumulation of wintering bees starts ($t_{15} \div t_{16}$).

Designations of variables and their actual values are described in Table 1.

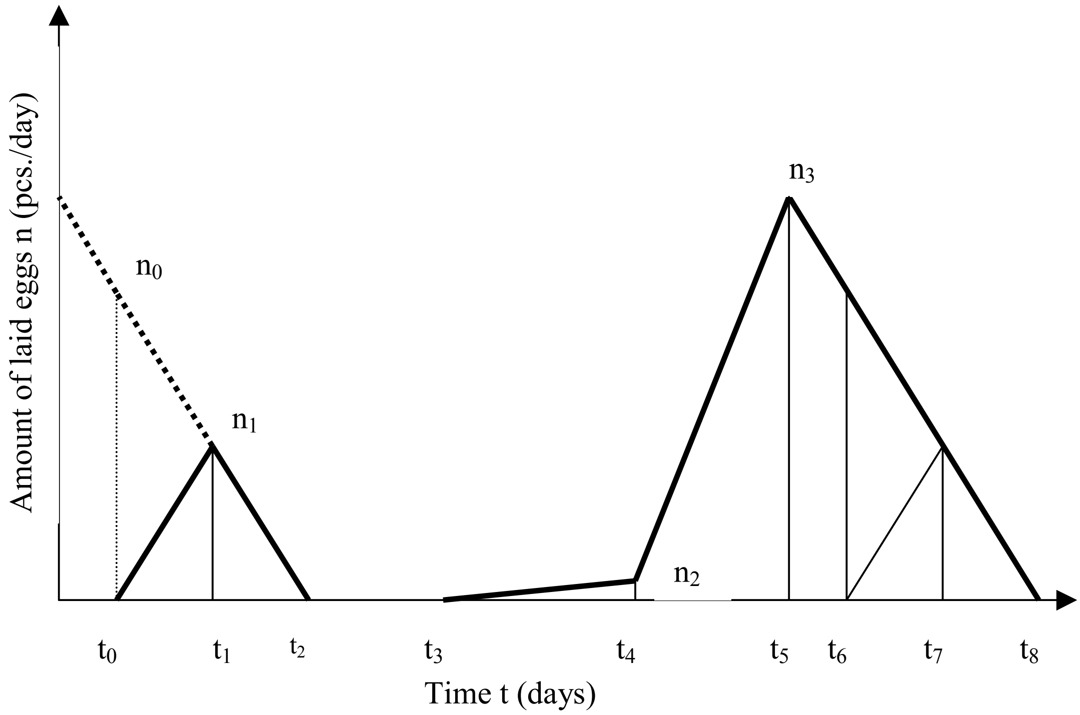

Figure 2. Curve of bee queen laying.

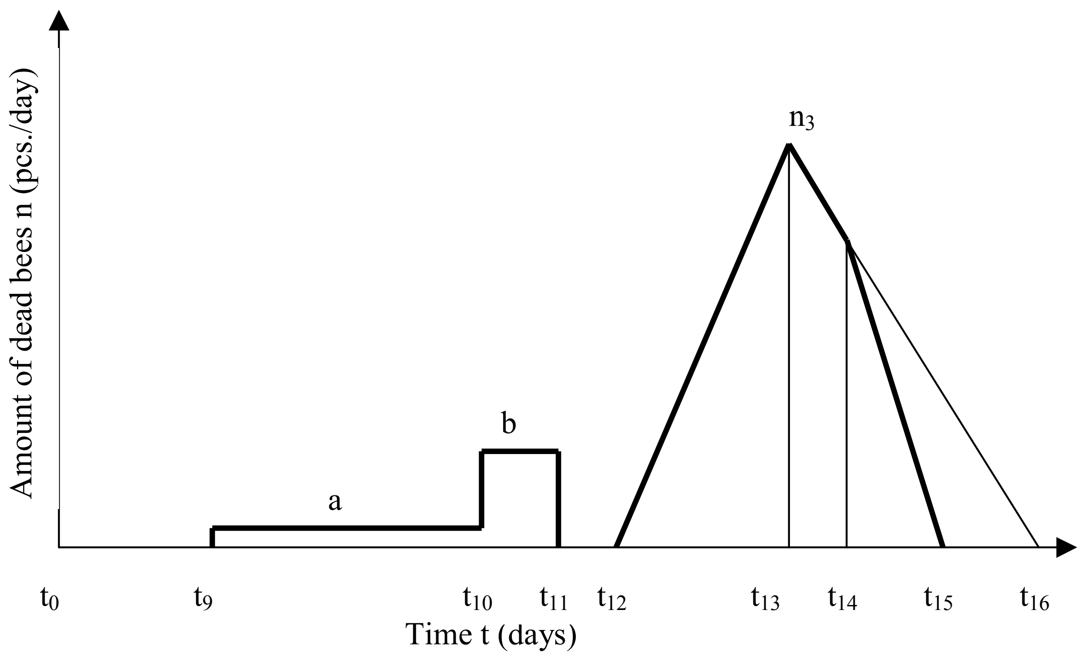

Figure 3. Bee death curve.

Table 1.

Designations of independent and dependent input parameters and their possible values in Latvian climatic circumstances.

No.	Des ign.	Description	Possible range of value or it's calculation art
Independent input parameters.			
1.	t_0	First egg for accumulation of wintering bees is laid	26.06.-06.07. 1-st year
2.	t_2	Stopping of egg laying in 1-st year	24.09.-04.10. 1-st year
3.	t_3	Start of egg laying in winter cluster	24.11.-06.12. 1-st year
4.	t_4	Start of intensive egg laying in spring	01.03.-15.04. 2-nd year
5.	t_5	Maximum of intensive egg laying in summer (n_3)	24.09.-04.10. 2-nd year
6.	t_6	First egg for accumulation of wintering bees is laid	26.06.-06.07. 2-nd year
7.	t_8	Stopping of egg laying in autumn	26.09.-06.10. 2-nd year
8.	n_1	Maximum of laid eggs for breeding of wintering bees	600-1000 eggs/day; 1-st year
9.	n_2	Maximal egg laying in winter cluster (t_4)	0-300 eggs/day
10.	n_3	Maximum of intensive egg laying in summer (t_5)	1000-2000 eggs/day; 2-nd year
11.	t_b	Period of death of wintering bees	40-60 days; 2-nd year
12.	t_v	Average bee lifespan in summer	30-40 days; 2-nd year
13.	a	Average bee loss in winter	5-15 bees/day

Dependent input parameters			
1.	t_1	Maximum of egg laying for accumulation of wintering bees (n_1)	$t_1=(t_2-t_0)/2$, 1-st year
2.	t_7	Maximum of egg laying for accumulation of wintering bees	$t_7=(t_8-t_6)/2$, 2-nd year
3.	t_9	Start of dying of bees in wintering period	$t_9=t_2+21$;
4.	t_{10}	Start of intensive dying of overwintered bees	$t_{10}=t_4+t_n$;
5.	t_{11}	End of intensive dying of overwintered bees	$t_{11}=t_{10}+t_b$;
6.	t_{12}	Start of bee losses in summer	$t_{12}=t_4+21+t_v$;
7.	t_{13}	Maximal bee loss in summer	$t_{13}=t_5+21+t_v$;
8.	t_{15}	End of bee losses in summer	$t_{15}=t_{14}+(t_{16}-t_{14})/2$;
9.	t_{16}	Seeming end of summer bee loss	$t_{16}=t_8+21+t_{v;}$
10.	t_{14}	Start of accumulation of wintering bees	$t_{14}=t_0+21+t_v$;
11.	n_0	Value of queen egg laying in the moment when the first egg for wintering bee is laid	$n_0=n_1/2t_1+n_2/2(t_4-t_3)]-a(t_{10}-t_9)$;
12.	N_0	Number of bees in spring (t_{10})	$N_0=n_1/2t_2+n_2/2(t_4-t_3)-a(t_{10}-t_9)$;
13.	b	Average loss of wintering bees during period of intensive death	$b= N_0/ t_b$;

To calculate the amount of born bees until day t starting from point $t_0=0$ the assumed egg laying curve has to be integrated and moved for 21 day (period of development from egg to bee). Equations necessary for these calculations are summarised in Table 2.

Table 2.

Equations to calculate amount of born bees until any day t.

Time (d)	**Amount of born bees until the day t**
$t<21$	$B=0$
$t\le t_2+21$	$B=\frac{n_1(t-21)^2}{2t_1};$
$t\le t_3+21$	$B=\frac{n_1}{2}[t_2-\frac{(t_2-t+21)^2}{t_2-t_1}];$
$t\le t_4+21$	$B=\frac{n_1t_2}{2};$
$t\le t_5+21$	$B=\frac{n_1t_2}{2}+\frac{n_2(t-t_3-21)^2}{2(t_4-t_3)};$
$t\le t_6+21$	$B=\frac{n_1t_2}{2}+\frac{n_2(t_4-t_3)}{2}+n_2(t-t_4-21)+\frac{(n_3-n_2)(t-t_4-21)^2}{2(t_5-t_4)};$
$t\le t_8+21$	$B=\frac{n_1t_2}{2}+\frac{n_2(t_4-t_3)}{2}+\frac{(n_2+n_3)(t_5-t_4)}{2}+\frac{n_3}{2}[t_8-t_5-\frac{(n_3-n_2)(t_8-t+21)^2}{t_8-t_5}];$
$t> t_8+21$	$B=\frac{n_1t_2}{2}+\frac{n_2(t_4-t_3)}{2}+\frac{(n_2+n_3)(t_5-t_4)}{2}+\frac{n_3(t_8-t_5)}{2};$

We have to use following equation to calculate amount of x days old bees in day t [$B_x(t)$]:

$$B_x(t) = B_{(t-x)} - B_{(t-x-1)}, \tag{2}$$

where, $B_{(t-x)}$ – amount of bees in day (t-x); $B_{(t-x-1)}$ - amount of bees in day (t-x-1).

We have to use following equation to calculate amount of bees in age interval x_1-x_2, [$B_{x2-x1}(t)$]:

$$B_{x2-x1}(t) = B_{(t-x2)} - B_{(t-x1-1)}, \tag{3}$$

where, $B_{(t-x2)}$ - amount of bees in day (t-x_2); $B_{(t-x1-1)}$ amount of bees in day (t-x_1-1).

Similar calculations are possible to calculate amount of brood of different age (eggs, larvae, pupa) in bee colony in any day.

The same way we can determine equations for calculations of dead bees (Table 3).

Table 3.

Equations to calculate amount of dead bees until any day t.

Time (d)	Amount of dead bees until the day t
$t<t_9$	N=0;
$t\le t_{10}$	$N=a(t-t_9)$;
$t\le t_{11}$	$N=a(t_{10}-t_9)+b(t-t_{10})$;
$t\le t_{12}$	$N=a(t_{10}-t_9)+b(t_{11}-t_{10})$;
$t\le t_{13}$	$N=a(t_{10}-t_9)+b(t_{11}-t_{10})+\frac{n_3(t-t_{12})^2}{2(t_{13}-t_{12})}$;
$t\le t_{14}$	$N=a(t_{10}-t_9)+b(t_{11}-t_{10})+\frac{n_3(t_{13}-t_{12})}{2}+\frac{n_3}{2}[t_{16}-t_{13}-\frac{(t_{16}-t)^2}{t_{16}-t_{13}}]$;
$t\le t_{15}$	$N=a(t_{10}-t_9)+b(t_{11}-t_{10})+\frac{n_3(t_{13}-t_{12})}{2}+\frac{(n_3+n_4)(t_{14}-t_{13})}{2}+\frac{n_4}{2}[t_{15}-t_{14}-\frac{(t_{15}-t)^2}{t_{15}-t_{14}}]$;
$t>t_{15}$	$N=a(t_{10}-t_9)+b(t_{11}-t_{10})+\frac{n_3(t_{13}-t_{12})}{2}+\frac{(n_3+n_4)(t_{14}-t_{13})}{2}+\frac{n_4(t_{15}-t_{14})}{2}$;

There is software created for the model using electronic tables Excel 97. Using computers with Pentium 2 processor time of calculations for one variant is about 5 seconds. To improve the overview results are presented in form of curves (Figure 4), besides there are two variants in the same picture. Starting the job the inputs for both variants should be equal. Changing one or more parameters in one of variants the difference between initial and new graph can be clearly determined.

Besides the changes in amount of bees in colony the dynamics of amount of eggs, larvae, pupa and bee groups of different age during the observed time period is calculated. Important is to calculate amount of bees in groups of different age because they are specialising depending on the age (Table 4).

Table 4.
Bee age doing different tasks

Task	Bee age, d.
Cleaning cells	1 – 3
Feeding	4 – 11
In hive (different)	12 – 21
Flying	22 – end of life

DISCUSSION AND RESULTS

Under natural circumstances bee queen egg laying is similar to triangle rule (McLellan et al., 1980). Using trapeze it is possible to forecast bee colony development under non-realistic circumstances which allows to estimate potential of bee colony in use of nectar yield. If that way there would be found a useful graphic of egg laying it would be possible to search way to practical fulfilling.

If in the chosen variant there is a situation during intensive spring development period t_4-t_5 when amount of brood is bigger than the one of bees calculations are stopped and it is suggested to change the input parameters so that the unrealistic situation would be eliminated. The limitation is based on a fact that each wintering bee is able to grow one new bee (Argalis et al., 1970).

In the model is not taken into consideration obstacle that a part of brood perishes during their development. The difference till 6% (Harris, 1985) we consider as insignificant and systematic which is not essentially influencing results of calculations. If the user considers it as important the egg laying characterising parameters have to be increased by 1.06 times.

The amount of wintering bees is determined by the amount of eggs lied by queen in the end span from which the wintering bees are developing. That is described in the model by isosceles-triangle t_0, n_1, t_2 or t_6, n_1, t_8, respectively in the 1-st and 2-nd year. That is based on experimental results (Ulanowski, 1987).

A sample of calculations is showed in the Figure 4. The obtained results of calculations are showing that character of changes of bees amount corresponds to experimental data (Greg et al, 1949), (Winston, 1992).

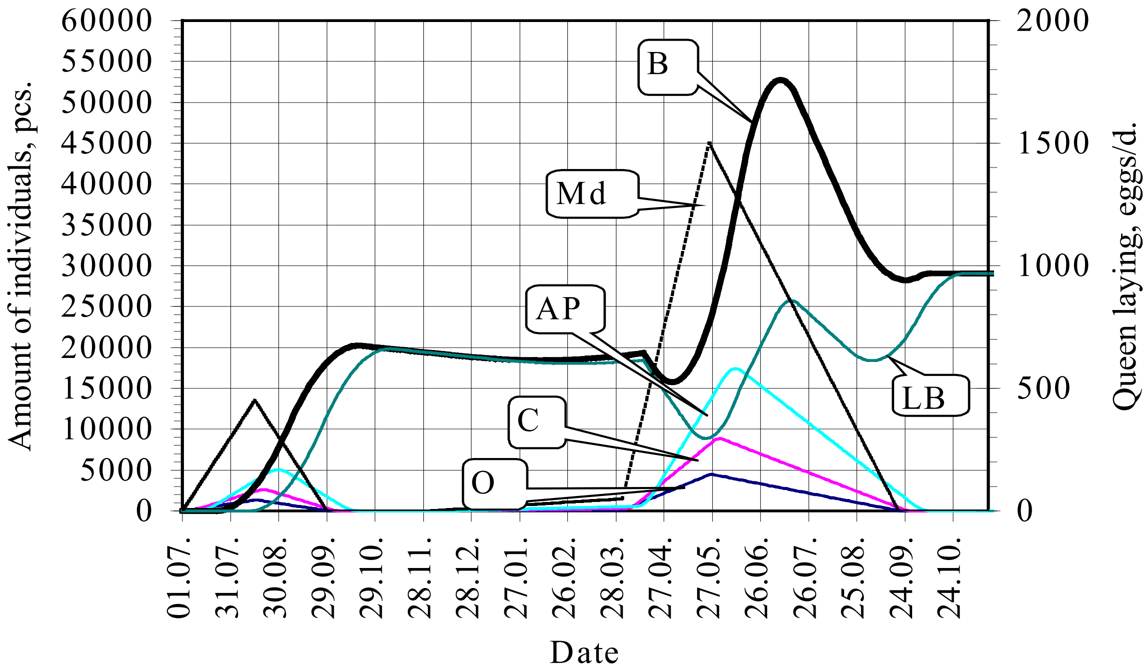

Figure 4. Sample of calculations where O-eggs (pcs); C-larvae (pcs); AP-pupa (pcs); B-bees (pcs); LB-flying bees (pcs) and Md-queen laying (eggs/d).

CONCLUSIONS

There is a linear bee colony development model worked out starting from period of accumulation of wintering bees in the 1-st year until start of wintering in the second year. To describe bee queen laying a trapeze is used (in special cases – triangle or rectangle). The model has 13 input parameters – time: t_0; t_2; t_3; t_4; t_5; t_6; t_8; t_b; t_v, bee loss a and characteristic of queen laying: n_1; n_2; n_3, and 13 dependent (calculated) parameters. Software is made on Excel 97 basis. Result is dynamics of amount changes of eggs, larvae, pupa, bees-cleaners, bees-feeders, flying bees and bees totally during the observed time period. Character of bees amount changes is similar to experimental results.

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